Cracking a Logical Puzzle with ROSETTA

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Abstract

This note briefly describes how ROSETTA can be used to solve a small logical puzzle.

1   Introduction

The ROSETTA toolkit [4, 5, 6, 2] for discernibility-based data analysis is usually employed for supervised learning purposes. However, ROSETTA can also be used as a general tool for solving logical puzzles of a certain kind. This document provides a small example of how this can be accomplished.

2   The Puzzle

Consider the following logical puzzle, taken from [9]:

> The police commissioner explained, "We're trying to crack a major drug cartel." Commissioner Bratt is well known for his work in bringing down New York crime. It's less well known that he has been a frequent visitor to Ecco's apartment over the years. "We're using informants, but these guys are rewarded for their information, so they tend to, well, dissemble."
>
> "That means 'lie,' doesn't it?" Liane asked with a smile.
>
> "Well, yes, young lady," Bratt said after a brief pause, "but these are the best we could get. This cartel is one of the smoothest and smartest we've ever seen. It's run out of Argentina, with offices all over Bolivia. You've met the chief, Ecco, I think," Bratt continued. "The professor here talked about Solaris (also known as El Jefito, because of his diminutive size) is his monograph Codes, Puzzles, and Conspiracy. You met him at the Casino in Punta del Este. You never figured out his role in Baskerhound's escapades, unfortunately."
>
> "Ah, yes, the Rhodes Scholar wrestler with a weakness for bimbos," Ecco said with a smile.
>
> "Right. Especially ones wearing tightfitting clothes," the Commissioner responded. "Some of those beauties figure among our informants."
>
> "Well, how many informants are there altogether?" Ecco asked. "Twenty," said the Commissioner. "I think he has turned some of them, but not most, I hope. The result, however, is that we now have some informants calling other ones liars. We don't know whom to trust. The thing is, we have vetted these informants carefully, so we think that not more than five have turned, though it may be as many as eight," Bratt said.
>
> "So, you know neither who lies, nor how many?" asked Liane in a slightly mocking tone.
>
> Bratt glared at her, but nodded. "That's right. It could be any of them."
>
> "Can you tell us who accuses whom?" Ecco asked.
>
> Bratt laid out a list (changing the names for obvious reasons):

Accuser	Accused
Petra	Gwenyth
Sam	Larry
Hillary	Petra
Olivia	Petra
Gwenyth	Alan
Dave	Mike
Isaac	Nick
Hillary	Kris
Sam	Jack
Sam	Rivera
Gwenyth	Mike
Olivia	Ulm
Jack	Larry
Jack	Sam
Alan	Larry
Isaac	Mike
Hillary	Ulm
Ulm	Larry
Gwenyth	Jack
Gwenyth	Olivia
Olivia	Kris
Larry	Mike
Tom	Hillary
Emily	Isaac
Petra	Rivera
Mike	Larry
Dave	Ulm
Hillary	Bob
Larry	Hillary
Gwenyth	Kris

"Well, Liane," Ecco said. "What do you make of the situation?"

"I need some more clarification," she replied. "Commissioner Bratt, am I correct in drawing the following inferences? Suppose X accuses Y of lying: At least one is a liar. If we know that X tells the truth, then Y is a liar. If we know that Y tells the truth, then X is a liar. If X lies, then his accusations may or may not be true."

"Absolutely our thinking," said the Commissioner, visibly impressed. "The last point is particularly important. Lying informants may still point the finger at other liars. Also, we don't fault truth-tellers for failing to accuse liars. They simply may not know enough."

"And our job," Liane went on, "is to find the smallest set of liars that explain all these accusations?"

"Right again," Bratt said, as he glanced anxiously at Ecco.

Ecco nibbled at a scone and stared at the list. Liane doodled while she looked at it.

After a few minutes, Ecco wrote eight names on a post-it and handed it to the Commissioner. "Here are a set of eight liars who could explain all these accusations. Liane, did I make a mistake? Can you solve the problem with fewer liars?"

"I'm not sure," Liane said. "There have to be at least six, though."

Reader, your puzzle: First, why do there have to be at least six? Second, are eight liars a sufficient number to explain all these accusations? If so, which eight? Could there be fewer? If so, who would they be?

3 The Solution

The puzzle can probably be solved in a number of different ways. The key to using ROSETTA to crack the puzzle lies in viewing it as a Boolean minimization problem [1].

Let (X_i, Y_i) be an accusation pair in Commissioner Bratt's table. According to Liane's and the Commissioner's reasoning, at least one of X_i and Y_i is a liar, possibly both. Then, there has to be at least eight liars because there are the following non-overlapping pairs:

Accuser	Accused
Gwenyth	Alan
Dave	Mike
Sam	Jack
Ulm	Larry
Olivia	Kris
Tom	Hillary
Emily	Isaac
Petra	Rivera

Next, we observe that the set of lying informants is defined through the list of accusations. Since at least one of the informants in each accusation pair (X_i, Y_i) must be a liar, we can construct a Boolean POS function f that expresses what the set of lying informants must be. This is done in Equation 1, where X_i^* and Y_i^* are Boolean variables corresponding to X_i and Y_i, respectively.

$$f = \prod_{i=1}^{30} (X_i^* + Y_i^*) \tag{1}$$

The crucial observation to make is that a minimal set of liars is determined by a prime implicant of f. These prime implicants can be computed by ROSETTA.

3.1 ROSETTA GUI Preliminaries

For this case study, the following ROSETTA GUI details are worth noting:

- An information system can be read into a new ROSETTA project by selecting **Open…** from the main **File** menu, and will be placed immediately below the root of the **Structures** node in the project tree.

- Branches in the project tree can be expanded or collapsed by left-clicking on the "⊞" or "⊟" symbols next to the icons.

- Right-clicking on an icon in the project tree brings up a pop-up menu for that object. In the following, the symbol "▶" will be used to denote menu navigation.

- Left-clicking twice on an icon in the project tree can be used as a shortcut for viewing that object in detail.

- To rename an object, first left-click once on its icon to select it. Then left-click once more on the icon's label. The icon's label is edited directly in place.

- To view progress messages and warnings, select **Messages** from the main **View** menu.

3.2 Required Steps

The steps required to solve the logical puzzle with ROSETTA are as follows: First, define f and import this definition into ROSETTA as a binary information system. Then, compute reducts relative to the first object in the table. The solutions are obtained by examining the resulting reduct set. For details on this process, see [3].

```
Liars =
   (petra + gwenyth) *
   (sam + larry) *
   (hillary + petra) *
   (olivia + petra) *
   (gwenyth + alan) *
   (dave + mike) *
   (isaac + nick) *
   (hillary + kris) *
   (sam + jack) *
   (sam + rivera) *
   (gwenyth + mike) *
   (olivia + ulm) *
   (jack + larry) *
   (jack + sam) *
   (alan + larry) *
   (isaac + mike) *
   (hillary + ulm) *
   (ulm + larry) *
   (gwenyth + jack) *
   (gwenyth + olivia) *
   (olivia + kris) *
   (larry + mike) *
   (tom + hillary) *
   (emily + isaac) *
   (petra + rivera) *
   (mike + larry) *
   (dave + ulm) *
   (hillary + bob) *
   (larry + hillary) *
   (gwenyth + kris)
end
```

Table 1: A definition of a POS function in ROSETTA format. ROSETTA can import this function as a binary decision system [3].

The above outlined procedure can be executed either through writing a ROSETTA script that automates the process, or by carrying out the steps manually in the ROSETTA GUI. Using the ROSETTA GUI, do the following, starting from a new project:

1. Create an ASCII file containing our Boolean function defined in a format that ROSETTA can read. This is done in Table 1. In the following, this file will be referred to as `liars.txt`.

2. Import the function into ROSETTA.

    P▶**Load…**

    Use the following parameters:

    ```
    FILENAME = liars.txt;
    ```

    The system will automatically detect that the file is in a non-native format, and query you for the file type. Specify ROSETTA POS function format. Upon success, a decision system will appear in the project tree.

3. Compute all reducts relative to the first object in the decision system.

    D▶**Reduce**▶**Exhaustive calculation (RSES)…**

    Use the following parameters:

    ```
    DISCERNIBILITY = Object;        SELECTION = Index;
    MODULO.DECISION = True;         SELECTION.INDEX = 0;
    ```

    Upon success, a set of reducts with an associated set of rules will appear in the project tree.

4. Solutions to the logical puzzle are obtained by viewing the set of reducts.

   R▶**View…**

   The prime implicants of the function can be sorted according to length by right-clicking the **Length** column in the grid view.

Our Boolean function has 42 prime implicants in all, i.e., there are 42 different minimal set of liars that can explain all the accusations that are made. We can obtain some statistics by invoking the following:

R▶**Statistics…**

Items in list views can be sorted by left-clicking the key column.

We see that we have 3 prime implicants of length 8, 5 of length 9, 15 of length 10, 11 of length 11, 6 of length 12 and 2 of length 13. It is the shortest prime implicants, i.e., those of length 8 that interest Commissioner Bratt. The minimal sets of liars that correspond to these are:

- {Petra, Gwenyth, Sam, Larry, Hillary, Olivia, Dave, Isaac}

- {Gwenyth, Larry, Hillary, Olivia, Dave, Isaac, Jack, Rivera}

- {Gwenyth, Sam, Larry, Hillary, Olivia, Dave, Isaac, Rivera}

It might be interesting to see how often each informant occurs across the set of possible minimal sets. This information could be used, e.g., by Commissioner Bratt to quantify the degree of belief he has in each of the informants being a liar. If an informant occurs in very many of the 42 possible minimal sets, then it might be perceived as likely that the informant is a liar. Conversely, if an informant occurs in very few of the 42 possible minimal sets, then the informant is likely to be a truth-teller. Examining our set of prime implicants in this way, we obtain the following ranking:

Informant	Count
Bob	10 out of 42
Tom	10 out of 42
Dave	14 out of 42
Alan	18 out of 42
…	…
Hillary	32 out of 42
Petra	32 out of 42
Ulm	32 out of 42
Larry	34 out of 42
Mike	35 out of 42

Acknowledgments

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